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PHYSICS OF SEMICONDUCTOR DEVICES

Improvement of the Efficiency of Silicon Solar Cells

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Abstract—The efficiency of a silicon solar cell can be greatly improved as a result of a drastic decrease in the dark current, which is attained by replacing the continuous heavily doped layers for a set of small heavily doped regions, the distances between which are much larger than their size. The effect is caused by the fact that the major contribution to the dark current is provided by surface recombination, which greatly increases with doping near the interface with the insulator. It is shown that there is an optimal relationship for the distances between regions and their sizes at which the dark current is considerably suppressed, but there is no appreciable decrease in the photocurrent due to deterioration in conditions of minority-carrier collection, and no significant series resistance is introduced. In this case, the operating voltage and efficiency of the solar cell substantially increase.

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1. INTRODUCTION

The efficiency of commercial solar cells on single-crystal Si under conventional conditions (spectrum AM 1.5, 0.1 W/cm^2, 25°C) has reached 19.5% [1]. Its further increase is hindered first of all by the lowered open-circuit voltage V_{oc}.

This follows from the estimate of V_{oc} and the short-circuit current I_{sc} according to the formulae

$$V_{oc} = (T/e)\ln(e\eta J/i_s), \quad (1a)$$

$$I_{sc} = -e\eta SJ, \quad (1b)$$

where J is the photon-flux density, η is the quantum yield of the device, S is its area, i_s is the saturation-current density, T is the temperature in units of energy, and e is the elementary charge. The current density $I_{sc}/S \approx 0.0377$ A/cm^2 [1] corresponds to the flux density of electrons $\eta J = 2.35 \times 10^{17}$ 1/cm^2 s. The flux density of photons with an energy larger than the silicon band gap 1.12 eV at AM 1.5 and 0.1 W/cm^2 according to ASTM G173-03 data [2] amounts to $J_G \approx 2.74 \times 10^{17}$ 1/cm^2 s. Therefore, $\eta J/J_G \approx 0.86$; i.e., only 14% of these photons give no contribution to the photocurrent.

Substituting $V_{oc} = 0.641$ V [1] in formula (la) and the value of ηJ, we have $i_s \approx 5.44 \times 10^{-13}$ À/cm^2. The saturation-current density i_{sb} caused by recombination in the substrate (base) of such a cell for the case of highly efficient photoconversion (at $d \ll L$, where d is the base thickness, $L = \sqrt{D\tau}$, D is the diffusivity, and τ is the lifetime) is

$$i_{sb} \approx en_i^2 d/N_a\tau, \quad (2)$$

where n_i is the intrinsic concentration of carriers, N_a is the acceptor concentration in the base (here is p-type conductivity). Formula (2) for the typical parameters ($d = 0.02$ cm, $N_a = 1.5 \times 10^{16}$ cm^{-3}, $\tau = 2 \times 10^{-4}$ s, and $n_i = 8.79 \times 10^9$ cm^{-3} at 25°C [3, 4]) gives $i_{sb} = 8.2 \times 10^{-14}$ A/cm^2. Hence, $i_{sb} \ll i_s$, and it is the recombination of carriers in low-resistivity areas that determines the value of i_s, i.e., a relatively high emitter saturation current, which decreases the voltage V_{oc}.

By suppressing the emitter saturation current, we can appreciably increase V_{oc} and the solar-cell efficiency, especially, with increasing lifetime. It is limited by Auger recombination. Its coefficient is $\gamma_A = 1.3 \times 10^{-30}$ cm^6/s at $N_a = 1.5 \times 10^{16}$ cm^{-3} [5] which gives $\tau = 3.4 \times 10^{-3}$ s, and $i_{sb} = 4.85 \times 10^{-15}$ À/cm^2. Then at $i_s \approx i_{sb}$ and $\eta J = 2.35 \times 10^{17}$ 1/cm^2 s, it follows from formula (1a) that $V_{oc} \approx 0.762$ V, which is 19% larger than the value from [1].

In this publication, we describe the method of suppression of this current. It is predominantly formed near the metal–semiconductor and (heavily doped semiconductor)–insulator interfaces, where the surface-recombination rate (SR) is high, and it can be suppressed by decreasing both the area of the contacts to the metal and the doping level near the interface with the insulator. This can be attained by the heavy doping of the small-area regions of "islands" instead of continuous layers. In this case, the collection of minority carriers becomes less efficient, and the photocurrent decreases. However, as is shown in what follows, there are optimal parameters of islands at which the saturation current drastically decreases, and photocurrent decreases only slightly (or does not decrease at all), which greatly improves the solar-cell efficiency.

Further, we first consider the flow of photocurrent and dark current in a conventional solar cell and, then, elucidate, how these processes and their characteristics vary in an island-structured solar cell. The entire analysis is carried out for the case of low-intensity injection.

The processes in the p-type substrate are described by the continuity equation for electrons

$$D\Delta n - (n - n_{p0})/\tau = -G(z), \tag{3}$$

where n is the electron concentration, n_{p0} is the equilibrium value of this concentration, and $G(z)$ is their rate of generation by the incident radiation dependent only on coordinate z. In Eq. (3), it is assumed that the flux density of electrons $\mathbf{j}$ is determined by their diffusion: $\mathbf{j} = -D\Delta n$.

2. CONVENTIONAL SOLAR CELL

Let us consider a solar cell with a p-type base located between planes $z = 0$ and $z = d$. In plane $z = 0$, there is an n^+ layer and a p–n junction, and, in plane $z = d$, there is only a p^+ layer. In this section, we consider their thickness as negligibly small in comparison with d and L, and Eq. (3) is solved in the one-dimensional approximation.

First, we calculate the photocurrent in the absence of voltage and, then, we calculate the dark current with voltage V application. The total current at $V \neq 0$ is the superposition of these currents because the equations to be solved are linear in the case of low-intensity injection.

2.1. Photocurrent

We calculate the photocurrent solving Eq. (3) with the boundary conditions

$$n|_{z=0} = 0, \tag{4a}$$

$$-D\partial n/\partial z|_{z=d} = s_d n|_{z=d}. \tag{4b}$$

In condition (4a), we take into account that the velocity of electrons generated in the base sharply increases in the field of the p–n junction, due to which their concentration near the junction becomes much lower than in the base depth (but very much exceeds n_{p0}). Condition (4b) expresses the electron flux at $z = d$ in terms of the effective surface-recombination ~~rate~~ (ESRR) s_d, which takes into account both Auger recombination in the p^+ layer conventionally formed near this interface and surface recombination at the interface of this layer with the metal.

The general one-dimensional solution of Eq. (3) at $n \gg n_{p0}$ has the form:

$$n(z) = C_1\exp(-z/L) + C_2\exp(z/L) - \frac{L}{D}\int_0^z d\xi G(\xi)\sinh\frac{z-\xi}{L}. \tag{5}$$

From formula (5) and boundary conditions (4), we find the expression for the electron concentration

$$n(z) = \frac{L}{D}\left[\sinh\frac{z}{L}\int_0^d d\xi G(\xi) \times\left(\cosh\frac{d-\xi}{L} + \frac{Ls_d}{D}\sinh\frac{d-\xi}{L}\right)\Big/\left(\cosh\frac{d}{L} + \frac{Ls_d}{D}\sinh\frac{d}{L}\right) - \int_0^z d\xi G(\xi)\sinh\frac{z-\xi}{L}\right],$$

and the expression for the density of photocurrent through the p–n junction

$$i_L = eD\frac{\partial n}{\partial z}\Big|_{z=0} = e\int_0^d d\xi G(\xi)\left(\cosh\frac{d-\xi}{L} + \frac{Ls_d}{D}\sinh\frac{d-\xi}{L}\right)\Big/\left(\cosh\frac{d}{L} + \frac{Ls_d}{D}\sinh\frac{d}{L}\right). \tag{6}$$

The solar cell is highly efficient if the recombination losses are low, i.e., for

$$d/L \ll 1, \tag{7a}$$

$$s_d \ll D/d. \tag{7b}$$

In this case, formula (6) can be simplified:

$$i_L \approx e\int_0^d d\xi G(\xi)\left(1 - \frac{1}{2}\frac{\xi}{L}\frac{2d-\xi}{L} - \frac{\xi s_d}{D}\right) \approx \int_0^d d\xi G(\xi). \tag{8}$$

2.2. Dark Current

To calculate the dark current, we use Eq. (3) written in the form

$$D\partial^2 n/\partial z^2 - (n - n_{p0})/\tau = 0, \tag{9}$$

together with the boundary conditions, replacing conditions (4) in this case

$$n|_{z=0} = n_{p0}\exp(eV/T), \tag{10a}$$

$$-D\partial n/\partial z|_{z=d} = s_d(n|_{z=d} - n_{p0}). \tag{10b}$$

Solving Eq. (9) together with boundary conditions (10a) and (10b), we obtain

$$n(z) = n_{p0}\left\{1 + [\exp(eV/T) - 1]\left(\cosh\frac{d-z}{L} + \frac{Ls_d}{D}\sinh\frac{d-z}{L}\right)/\left(\cosh\frac{d}{L} + \frac{Ls_d}{D}\sinh\frac{d}{L}\right)\right\}. \quad (11)$$

From here, the expression for the density of the electron dark current follows:

$$i_{nd} = eD\partial n/\partial z|_{z=0} = -e(n_i^2/N_a)\frac{D}{L}[\exp(eV/T) - 1] \times\left(\tanh\frac{d}{L} + \frac{Ls_d}{D}\right)/\left(1 + \frac{Ls_d}{D}\tanh\frac{d}{L}\right). \quad (12)$$

Here it is taken into account that $n_{p0} = n_i^2/N_A$. For highly efficient solar cells, where conditions (7a) and (7b), are fulfilled, we find from formulae (11) and (12), that $n(z) \approx (n_i^2/N_A)\exp(eV/T)$ and

$$i_{nd} \approx -e(d/\tau + s_d)(n_i^2/N_A)[\exp(eV/T) - 1]. \quad (12a)$$

In this case, the saturation-current density is the sum of the saturation base current density i_{sb} and the saturation ~~rear~~-contact current density $i_{sbc} \approx en_1^2 s_d/N_A$. Here, we disregard the recombination of holes at the face surface of the semiconductor, in the n^+ layer adjoining it, and inside the p–n junction, i.e., at $z = 0$. By designating the hole dark current density i_{pd} and introducing the ESR~~R~~ s_{0p} in terms of the saturation current density of the face contact i_{sfc} as $i_{sfc} = en_i^2 s_{0p}/N_A$, we obtain the expression for the total dark-current density:

$$i_d = i_{nd} + i_{pd} = -e(d/\tau + s_d + s_{0p})(n_i^2/N_A)[\exp(eV/T) - 1], \quad (13)$$

and, from it, the expression for the total saturation current density and for the effective recombination ~~rate~~ s_{eff} in the solar cell:

$$i_s = i_{sfc} + i_{sb} + i_{sbc} = e(d/\tau + s_d + s_{0p})(n_i^2/N_A) = es_{\text{eff}}(n_i^2/N_A).$$

From here, we find for the solar cells from [1] that $s_{\text{eff}} = 660$ cm/s and $s_{\text{eff}}\tau/d \gg 1$ {here we used the same parameters as previously for estimating from Eqs. (1a), (1b), and (2)}.

3. ISLAND-STRUCTURED SOLAR CELL (ISLANDS FROM THE FACE SIDE)

We replace the n^+ layer with n^+ islands arranged with the periods a and b along axes X and Y. We assume that the island surface (this means also the p–n-junction interface) is a hemisphere of radius $r_0 < a, b$. Further, we solve Eq. (3) in this configuration.

3.1. Photocurrent

Let us find the photocurrent collected from the area $A = ab$ by an island with a center at $r = 0$, where $r = \sqrt{x^2 + y^2 + z^2}$. At the p^+ contact, former condition (4b) is valid. Because there is no normal minority-carrier flux on the lateral surfaces, we have from here:

$$\partial n/\partial x|_{x = a/2, -a/2} = 0, \quad (14a)$$

$$\partial n/\partial y|_{y = b/2, -b/2} = 0, \quad (14b)$$

Boundary condition (4a) in this configuration is replaced for two conditions:

$$\text{at } r = r_0, \quad n|_{r = r_0} = 0, \quad (15a)$$

$$\text{at } z = 0, \quad r > r_0, \quad D\partial n/\partial z|_{z=0} = s_{0n}n|_{z=0}. \quad (15b)$$

Condition (15a), as well as (4a), reflects a sharp decrease in the concentration of carriers near the p–n junction in comparison with their concentration in the base depth, while condition (15b) describes the electron flux on the face surface of the semiconductor for the ESR~~R~~ s_{0n}.

The problem with boundary conditions (15a) and (15b) belongs to the class of incorrectly formulated problems. We solve it by applying the physical approach, which uses large and small parameters: it is assumed that the radius r_0 is very small and the diffusion length L is very large:

$$r_0 \ll a, b, d \ll L, \quad (16)$$

We show that, in this case, the saturation current of a solar cell can be sharply reduced with preservation of high light-conversion efficiency.

Under conditions (16), the electron concentration is homogeneous in the plane XY almost over the entire base (see further) except for small areas of a radius of several r_0 around the n^+ islands. Therefore, almost for the entire base, this concentration is described by expression (5) with boundary condition (4b).

To derive the second boundary condition, which replaces condition (4à), we consider the processes in the base at distances from the n^+ island, which are small in comparison with a, b, and d. Since the generation and recombination of electrons in this small region only slightly affect the electron flux, they can be disregarded in Eq. (3). The surface-recombination contribution is small in this region, which enables us to

replace condition (15b) with $\partial n(z)/\partial z|_{z=0} = 0$. In this case, our problem acquires spherical symmetry and Eq. (3) takes the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right) = 0. \quad (17)$$

Its solution at $r \geq r_0$ satisfying condition (15à) has the form

$$n(r) = C(1/r_0 - 1/r), \quad (18)$$

where C is an arbitrary constant. The electron flux J_{n^+} flowing in the p–n junction is

$$J_{n^+} = -2\pi r_0^2 D\partial n(r)/\partial r|_{r=r_0} = -2\pi CD. \quad (19)$$

We now study the behavior of the electron concentration and flux at values of r in the range

$$r_0 \ll r \ll a, b, d. \quad (20)$$

The electron concentration at such r only slightly depends on the distance to the nearest n^+ island {see Eq. (18)}, i.e., is almost homogeneous in the plane XY. Therefore, for $z \geq r$, it is well described by formula (5) and, because such r is much smaller than the characteristic lengths in formula (5), the electron concentration and flux density for them are close to the values of $n(0)$ and $j(0) = -D\partial n(z)/\partial z|_{z=0}$ following from formula (5). This flux comes to the n^+ island and leads to surface recombination. The bulk recombination here is small due to the right-hand inequality (16). Therefore, $j(0) = J_{n^+}/A + s_{0n}n(0)$ because the flux from area A flows to one n^+ island. Since we have $J_{n^+} = -2\pi Dr_0 n(0)$ from formulae (18) and (19), the second boundary condition for formula (5) replacing formula (4a) can be deduced from here as

$$-D\partial n(z)/\partial z|_{z=0} = -(2\pi Dr_0/A + s_{0n})n(0). \quad (21)$$

From formulae (5), (4b), and (21), we obtain the expression for the electron concentration

$$n(z) = \frac{L}{D}\frac{\left[L\left(\frac{2\pi r_0}{A}+\frac{s_{0n}}{D}\right)\sinh\frac{z}{L}+\cosh\frac{z}{L}\right]\int_0^d d\xi G(\xi)\left(\cosh\frac{d-\xi}{L}+\frac{s_d L}{D}\sinh\frac{d-\xi}{L}\right)}{L\left(\frac{2\pi r_0}{A}+\frac{s_{0n}+s_d}{D}\right)\cosh\frac{d}{L}+\left(1+\frac{\tau s_{0m}s_d}{D}+\frac{2\pi r_0\tau s_d}{A}\right)\sinh\frac{d}{L}} - \frac{L}{D}\int_0^z d\xi G(\xi)\sinh\frac{z-\xi}{L},$$

from which we find the photocurrent $I_{Li} = eA[D\partial n(z)/\partial z|_{z=0} - s_{0n}n(0)]$ for the n^+ island:

$$I_{Li} = \frac{eA\int_0^d d\xi G(\xi)[\cosh(d-\xi)/L] + (s_d L/D)\sinh[(d-\xi)/L]}{[1 + A(s_{0n}+s_d)/2\pi r_0 D]\cosh(d/L) + [Ls_d/D + A(1+\tau s_{0n}s_d/D)/2\pi r_0 L]\sinh(d/L)}. \quad (22)$$

Comparing formula (22) with formula (6), we find that the photocurrent density in the one-dimensional problem and the photocurrent I_{Li} of the n^+ island are related as follows:

$$I_{Li} = Ai_L/\{1 + (a/2\pi r_0 L)[L(s_{0n}+s_d)/D + (1+\tau s_{0n}s_d/D)\tanh(d/L)]/[1 + (Ls_d/D)\tanh(d/L)]\}. \quad (23)$$

For highly efficient solar cells {under conditions (7)}, expression (23) can be simplified:

$$I_{Li} = Ai_L/[1 + A(d/\tau + s_{0n} + s_d)/2\pi r_0 D]. \quad (24)$$

From here, it follows that the island-structured solar cell is highly efficient under conditions (16) if the island radius is within the range

$$A(d/\tau + s_{0n} + s_d)/2\pi D \ll r_0 \ll a, b, d. \quad (25)$$

3.2. Dark Current

Here we solve Eq. (3) for $G(z) = 0$ with the boundary conditions from the previous section with the only difference being that, if the voltage V is applied, it is necessary to replace boundary condition (15a) on the surfaces of the n^+ island for the following:

$$\text{for } r = r_0, \quad n|_{r=r_0} = n_{p0}\exp(eV/T). \quad (26)$$

In an efficient solar cell {see Eq. (7)}, the recombination rate is low and the concentration of injected carriers is almost homogeneous: $n(x, y, z) \approx n_{p0}\exp(eV/T)$ {see (11)}. Therefore, the dark current I_{di} flowing through the n^+ island is {compare with (13)}

$$I_{di} \approx -eA(d/\tau + s_{0n} + s_d + 2\pi r_0^2 s_{0pM}/A) \times n_{p0}[\exp(eV/T) - 1], \quad (27)$$

where s_{0pM} is the ESRR of holes in metal, and it is taken into account that the recombination of electrons proceeds near almost the entire face surface of the solar cell, while that of holes takes place only inside the n^+ islands. This sharply reduces the contribution of their recombination despite its very high rate.

In APPENDIX, these results are generalized to the case of an ellipsoidal surface of the n^+ island. If its second principal radius is βr_0, 2π in formula (24) should be replaced with $f(\beta)$ {see formulae (A. 9) and (A. 11)}, and 2π in Eq. (27) should be replaced with $\pi g(\beta)$ {see (A. 14), (A. 15)}. If the surface is flat (i.e., $\beta = 0$), $f(0) = 4$ and $g(0) = 1$.

4. BASIC CHARACTERISTICS OF SOLAR CELLS

We now compare the properties of conventional and island-structured cells with an identical area S. As follows from formulae (8), (13), (24), and (27), their currents I and I_I are as follows:

$$I = Si_L - eS(d/\tau + s_{0p} + s_d)n_{p0}[\exp(eV/T) - 1], \quad (28)$$

$$I_I = Si_L/\left[1 + \frac{A}{2\pi r_0 D}\left(\frac{d}{\tau} + s_{0n} + s_{dI}\right)\right] - eS\left(\frac{d}{\tau} + s_{0n} + s_{dI} + \frac{2\pi r_0^2}{A}s_{0pM}\right)n_{p0}(e^{eV/T} - 1). \quad (29)$$

In formula (29), the designation s_{dI} replaces s_d. Disregarding corrections for the order of magnitude of i_s/i_L, we use formulae (28) and (29) to find the relations between the short-circuit currents $I_{sc} = Si_L$ and I_{Isc}, and between the open-circuit voltages $V_{oc} \approx (T/e)\ln\{i_L/[e(d/\tau + s_d + s_{0p})n_{p0}]\}$ and V_{Ioc}:

$$I_{Isc} = I_{sc}/[1 + A(d/\tau + s_{0n} + s_{dI})/2\pi r_0 D], \quad (30)$$

$$V_{Ioc} = V_{oc} + \frac{T}{e}\ln\left[\frac{1}{1 + A(d/\tau + s_{0n} + s_{dI})/2\pi r_0 D} \times \frac{d/\tau + s_d + s_{0p}}{d/\tau + s_{dI} + s_{0n} + 2\pi r_0^2 s_{0pM}/A}\right]. \quad (31)$$

Thus, at

$$d/\tau + s_{0n} + s_{dI} + 2\pi r_0^2 s_{0pM}/A \ll d/\tau + s_d + s_{0p}$$

it is possible to significantly increase the open-circuit voltage. It follows from formulae (28)–(31) that the powers $P = IV$ produced by these solar cells attain peaks P_m and $P_{I,m}$ at voltages of V_m and $V_{I,m}$, and the currents I_m and $I_{I,m}$, determined by the expressions

$$V_m + (T/e)\ln(1 + eV_m/T) = V_{oc}, \quad (32a)$$

$$V_{I,m} + (T/e)\ln(1 + eV_{I,m}/T) = V_{Ioc}, \quad (32b)$$

$$I_m = I_{sc}/(1 + T/eV_m), \quad (33a)$$

$$I_{I,m} = I_{Isc}/(1 + T/eV_{I,m}). \quad (33b)$$

Let us find the radius $r_{0,\,opt}$ at which the power $P_{I,m}(r_0)$ is highest. Equating to zero its derivative with respect to r_0 and using formulae (32b) and (33b), we obtain the condition

$$d\ln(I_{Isc})/dr_0 V_{I,m} + dV_{Ioc}/dr_0 = 0.$$

From here and from formulae (30) and (31), we find the relation of $r_{0,\,opt}$ with other parameters of the solar cell (further, we designate $V_{I,m}(r_{0,\,opt}) \equiv V_{opt}$, $I_{I,m}(r_{0,\,opt}) \equiv I_{opt}$, and $P_{opt} = V_{opt}I_{opt}$):

$$\frac{A}{4\pi r_{0,\,opt}}\left(\frac{d}{r} + s_{dI} + s_{0n}\right)\left(\frac{eV_{opt}}{T} - 1\right) \times \left[1 + \frac{A}{2\pi r_{0,\,opt}^2 s_{0pM}}\left(\frac{d}{\tau} + s_{dI} + s_{0n}\right)\frac{eV_{opt} + T}{eV_{opt} - T}\right] = 1. \quad (34)$$

To calculate the values of V_{opt} and I_{opt}, we introduce the variables u and U:

$$u = \frac{A(d/\tau + s_{dI} + s_{0n})}{4\pi D r_{0,\,opt}}\left(\frac{eV_{opt}}{T} - 1\right), \quad (35a)$$

$$U = \frac{As_{0pM}(d/\tau + s_{dI} + s_{0n})}{8\pi D^2}\frac{(eV_{opt}/T - 1)^3}{eV_{opt}/T + 1}. \quad (35b)$$

In such variables, Eq. (34) takes the form (with $0 \le u < 1$):

$$U = u^3/(1 - u). \quad (36)$$

With the help of formulae (30)–(33), (35), and (36), we express V_{opt} and I_{opt} in terms of the variable u:

$$I_{opt} = I_{sc}/\left(1 + \frac{T}{eV_{opt}}\right)\left(1 + \frac{2u}{eV_{opt}/T - 1}\right), \quad (37)$$

$$\frac{V_{opt} - V_m}{T/e} + \ln\left(1 + \frac{V_{opt} - V_m}{V_m - T/e}\right) = \ln\left(\frac{d/\tau + s_d + s_{0p}}{d/\tau + s_{dI} + s_{0n}}\frac{1}{1 + 2uT/(eV_{opt} - T)} \times \frac{1}{1 + [u/(1 - u)][(eV_{opt} + T)/(eV_{opt} - T)]}\right). \quad (38)$$

From formulae (37) and (38), it can be seen that the values of V_{opt} and I_{opt}, and with them also the power P_{opt}, increase with decreasing u. The value of P_{opt} attains a peak P_{lim} at $u = 0$:

$$P_{lim} = V_{lim}I_{sc}/\left(1 + \frac{T}{eV_{lim}}\right) = P_m\frac{V_{lim}}{V_m}\left(1 + \frac{T}{eV_m}\right)/\left(1 + \frac{T}{eV_{lim}}\right), \quad (39)$$

where the value of V_{lim} is found from the solution of the equation

$$\frac{V_{\text{lim}} - V_m}{T/e} + \ln\left(1 + \frac{V_{\text{lim}} - V_m}{V_m + T/e}\right) = \ln\left(\frac{d/\tau + s_d + s_{0p}}{d/\tau + s_{dI} + s_{0n}}\right). \quad (40)$$

It follows from formulae (35) and (36) that the optimal radius and area are

$$r_{0,\text{opt}} = \frac{u^2}{1-u}\frac{2DT(eV_{\text{opt}} + T)}{s_{0pM}(eV_{\text{opt}} - T)^2}, \quad (41a)$$

$$A = \frac{u^3}{1-u}\frac{8\pi D^2 T^2(eV_{\text{opt}} + T)}{s_{0pM}(d/\tau + s_{dI} + s_{0n})(eV_{\text{opt}} - T)^3}. \quad (41b)$$

Therefore, at $u = 0$, $r_{0,\text{opt}} = 0$, $A = 0$, and the smaller the chosen radius of the n^+ island, the smaller u and A are. From formulae (39) and (40), the following equality is also obtained:

$$\frac{2\pi r_{0,\text{opt}}^2 s_{0pM}}{A(d/\tau + s_{dI} + s_{0n})} = \frac{u(1 + T/eV_{\text{opt}})}{(1-u)(1 - T/eV_{\text{opt}})}.$$

Since the left-hand side of the equality is the ratio of recombination currents inside and outside the n^+ island, its right-hand side shows that the contribution of the recombination inside the n^+ island is small for $u \ll 1$.

5. EFFECTIVE ~~RATES~~ OF SURFACE RECOMBINATION

At present, the major factor restricting an increase in the efficiency of solar cells is high ESR~~R~~s s_{0p} and s_d. They are high due to the fact that the SR~~R~~s s_f and s_b are very high at the face and ~~rear~~ surfaces of a semiconductor. Although, the approach of minority carriers to these surfaces is hindered due to the potential barriers, $s_{0p} \ll s_d$ and $s_d \ll s_b$, this proves to be insufficient. For a further decrease in s_{0p} and s_d, it is necessary to drastically decrease s_f and s_b. In what follows, we discuss the possibilities of such a decrease.

The ESR~~R~~ is highest at the interface with the metal. This is not important for the face surface because the metal covers only a small fraction of it (~4% of the cells with an efficiency of 19% [6]), while its ~~rear~~ surface is conventionally completely coated with the metal. At such an interface, the SR~~R~~ s_b attains 10^7 cm/s [7]. Near the interface with the metal, the p^+ layer pronouncedly decreases the ESR~~R~~ s_d in comparison with s_b. However, it is insufficient. For example, $s_d = 400$ cm/s [6] and $s_{\text{eff}} = 600$ cm/s in the cells with an efficiency of 19% (see Section 2); i.e., it is a high ESR~~R~~ at the interface with the metal that is the main restriction for their efficiency. To decrease s_d, an insulator is deposited onto the rear surface and atop of it, a metal in contact with the p^+ layer only in narrow slots [6]. Then the main recombination proceeds at the interface with the insulator, s_d decreases to ~150 cm/s, and the cell efficiency appreciably increases [6]. However, the fact that the SR~~R~~ $s_b \approx$ 45000 cm/s is very high at this interface [6] prevents a stronger decrease in s_d.

The matter is that the SR~~R~~ significantly increases with increasing level of silicon doping near the interface with the insulator. For example, in [7], the SR~~R~~ varied from 200 to 1000 cm/s for a phosphorus concentration of ~7.5×10^{18} cm^{-3} at the surface depending on the passivation method, attained (2–3) × 10^4 cm/s for ~2×10^{20} cm^{-3}, and only slightly depended on the passivation method. In [3] the SR~~R~~ is 300 cm/s for a phosphorus concentration of 1.5×10^{18} cm^{-3} at the surface and attained ~2×10^4 cm/s for ~5×10^{19} cm^{-3}. In the samples with a boron or phosphorus concentration of ~3×10^{15} cm^{-3} and different passivation methods, the SR~~R~~ varied from 10 to 100 cm/s [8] and amounted to ~10 cm/s in Si:B at a boron concentration of ~10^{16} cm^{-3} [9]. For light doping, the SR~~R~~ increases on a textured surface, but this effect disappears for heavy doping [3].

Explanation of these facts follows from the presence of local states of electrons and holes generated due to fluctuations in the charge built into the insulator at the semiconductor–insulator interface. The theory of such states [10, 11] made it possible to describe the properties of surface states at the Si:SiO_2 interface proceeding from the densities of positively and negatively charged centers $\Sigma_+ \sim \Sigma_- \sim 10^{12}$ cm^{-2} typical of SiO_2 [12]. A fluctuation with a positive charge binds an electron, and that with a negative charge, a hole. The larger the fluctuation charge, the higher the binding energy, while the probability of its formation is lower and increases with Σ_+ and Σ_-.

Therefore, the densities of such states decrease upon penetration into the band-gap depth and increase with Σ_+ and Σ_-. States with a binding energy up to ~(0.3–0.4) eV are formed by Gaussian fluctuations in the density of centers [10]. At a higher binding energy, they are formed by hybrid fluctuations with a Poisson (non-Gaussian) cluster of attracting centers within an attracting Gaussian fluctuation [11, 13]. The size of the Poisson fluctuation is small, and the electron (or hole) is bound to it quantum-mechanically. The size of the Gaussian fluctuation is large, and it classically deepens the quantum level. With increasing binding energy, the fraction of the classical shift in it decreases, and the fraction of the quantum localization increases. The density of such hybrid states at the Si:SiO_2 interface amounts to ~(10^{10}–10^{11}) eV^{-1} cm^{-2} at a binding energy of ~0.55 eV and decreases with increasing energy gradually due to an increase in the Poisson-fluctuation contribution [11]. Since these

states are generated at attracting centers, they capture, rapidly, only carriers from their allowed band.

For recombination, a carrier should tunnel between such electron and hole states lying at different points of the interface [14] and, so that this process is not too slow, the difference between the sum of their binding energies ε_{be} and ε_{bh} from the silicon band gap ε_G should not exceed the optical-phonon energy. Then this is a direct or indirect single-phonon process, and its time is the product of a very short time (on the order of $\hbar/\varepsilon_b$ for direct tunneling and on the order of the time of electron–phonon scattering for indirect tunneling, (see [11, 14]) and a large exponential component reflecting a low tunneling probability because its characteristic length is small in comparison with the distance between electron and hole states. It is clear that this exponent increases, and the SRR decreases with a decrease in the densities Σ_+ and Σ_- (with an improvement in the quality of the interface), the lower these densities, the more abrupt the changes. Recombination near the surface is always significant due to the presence of high densities of the electron and hole bound states.

From above, the explanation of the regularities of SRRs described in [3, 7–9] follows. For example, the higher the densities Σ_+ and Σ_-, and with them also the density of surface states, the higher the SRRs. This is the cause of the dependence of the SRR on the passivation method [8]. The densities Σ_+ and Σ_- depend on the surface orientation and are lowest for the [100] orientation [10]. Therefore the SRR is lower on the [100] surfaces than that on textured surfaces [3].

This is valid, while the impurity concentration N_S near the semiconductor surface is such that the average distance between impurity atoms is larger than the distance between the charges in the insulator, i.e., for $N_S < \Sigma^{3/2} \sim 10^{18}$ cm^{-3}. Such doping affects only slightly the SRR. If the semiconductor is heavily doped, for example, with donors, so that $N_S \gg 10^{18}$ cm^{-3}, it has a very high density of deep electron states generated mainly by the Poisson clusters of donors [16]. Near the interface with SiO_2, their density is higher than in the bulk because the potential of a charged center in the Si bulk is $e/\varepsilon_s r$, while it is $e/(\varepsilon_s + e/\varepsilon_i)r$ at the Si/SiO_2 interface (here, $\varepsilon_s \approx 12$ and $\varepsilon_i \approx 4$ are the permittivities of Si and SiO_2); i.e., it is 1.5 times larger than in the bulk. In this case, the surface-state density for electrons generated by donor clusters is much higher than for those created by fluctuations in density of charged centers in the insulator.[1] Therefore, the distance between the electron and hole states is drastically decreased, which leads to a very large increase in the tunneling probability and to a decrease in the tunneling time and a sharp increase in the SRR. In this case, the tunneling time ceases to depend on the distances between the charges built into the insulator, and the dependence of the SRR on the orientation of the semiconductor surface practically disappears.

[1] It should be noted that the density of the electron and hole surface states created by centers built into the insulator decreases with increasing doping because electron screening suppresses the contribution of large-scale fluctuations to the binding energy.

6. ISLAND-STRUCTURED SOLAR CELL (ISLANDS ON BOTH SIDES)

The efficiency of an island-structured solar cell increases with decreasing recombination rate $d/\tau + s_{dI} + s_{0n}$ {see formulae (38), (40)}. The rate can be reduced to ~1 cm/s because the semiconductor is not heavily doped at this interface, but in addition, it is possible to form here a potential barrier for electrons, for example, building a negative charge into the insulator. The rate d/τ for $d = 0.02$ cm can be reduced to 6 cm/s by increasing τ to 3.4×10^{-3} s (see Section 1). Another matter is the ESRR s_di. It cannot be reduced to a value smaller than ~150 cm/s because the semiconductor is doped to ~$(10^{20}$–$10^{21})$ cm^{-3} near this surface (see Section 5).

However, it is possible to greatly reduce s_{dI} by fabricating contact islands also on the rear side. To this end, a thin layer with a hole concentration of $p^+ \approx (1$–$3) \times 10^{18}$ cm^{-3} is formed near the rear surface of the base onto which the insulator is first deposited and, then, onto it, the metal in contact with this layer through narrow "windows", where the hole concentration p^{++} is reasonably high, (~10^{20}–10^{21}) cm^{-3} so that the resistance of the contact is low. For $p^+ \sim 10^{18}$ cm^{-3}, the formula $s_{dI} \approx s_b N_a/p^+$ is valid because the narrowing of the band gap and Auger recombination in the thin p^+ layer are small for such a concentration. Since the rate s_b falls within the range of 100–300 cm/s for $p^+ \sim 10^{18}$ cm^{-3} (see Section 5), s_{dI} can be reduced to values of ~1 cm/s, and the recombination rate $d/\tau + s_{dI} + s_{0n}$ can be reduced to ~10 cm/s.

The series resistance of such a structure can be made low with only a slight reduction in the efficiency of the solar cell. This can be seen from the following analysis. We assume that the p^{++} islands have a radius of r_b and follow along the axes X and Y with the period d, the conductivity of the p^+ layer is σ (measured in Ω^{-1}), and the current density in a working solar cell is i_m. One island collects the current from an area with a side of d and with the center coinciding with that of the island. This occurs due to generation of the field $E(r, \varphi)$ in the p^+ layer directed along this layer to the island center along the radius r. Here, φ is the angle around this center ($0 \le \varphi \le 2\pi$). We find this field equating the current in the layer flowing through the portion of $rd\varphi$ in length with the solar-cell current per area of a trapezium with the bases $rd\varphi$, $R(\varphi)d\varphi$, and the height $R(\varphi) - r$, where $R(\varphi)$ is the boundary of the area ($R(\varphi) = d/2\cos\varphi$ at $\pi/4 \le \varphi \le 3\pi/4$ and $3\pi/4 \le \varphi \le$

Optimized characteristics of island-structured solar cells for several values of parameters $r_{0,\,opt}$ and $d/\tau + s_{dI} + s_{0n}$

$d/\tau + s_{dI} + s_{0n}$, cm/s →	10			20			30		
$r_{0,\,opt}$, μm	A, cm^2	V_{opt}, V	η, %	A, cm^2	V_{opt}, V	η, %	A, cm^2	V_{opt}, V	η, %
5	1.86×10^{-4}	0.642	22.36	9.46×10^{-5}	0.625	21.76	6.36×10^{-5}	0.615	21.41
10	5.00×10^{-4}	0.639	22.10	2.54×10^{-4}	0.622	21.51	1.71×10^{-4}	0.612	21.16
15	8.83×10^{-4}	0.636	21.92	4.49×10^{-4}	0.619	21.33	3.02×10^{-4}	0.609	20.98

$5\pi/4$, $R(\varphi) = d/2\sin\varphi$) at $\pi/4 \le \varphi \le 3\pi/4$ and $5\pi/4 \le \varphi \le 7\pi/4$. It follows then that

$$E(r, \varphi) = [R^2(\varphi)/r - r]i_m/2\sigma. \quad (42)$$

Therefore, the power lost in an area with the side d of this layer is

$$P_l = \int_0^{2\pi} d\varphi \int_{r_b}^{R(\varphi)} dr r\sigma E^2(r, \varphi) \approx (i_m^2 d^4/6\sigma)\ln(d/2r_b),$$

and its ratio to the power $P_L = i_m V_m d^2$ produced here amounts to

$$P_l/P_L \approx (i_m d^2/6\sigma V_m)\ln(d/2r_b). \quad (43)$$

We now estimate these losses. At a p^+-layer thickness of 2 μm, $p^+ = 2 \times 10^{18}$ cm^{-3}, $\mu = 100$ cm^2/V s with $\sigma = 6.4 \times 10^{-3}\ \Omega^{-1}$. Then, for $V_m = 0.6$ V, $i_m = 0.04$ A/cm^2, $d = 200$ μm, and $r_b = 10$ μm from formula (43), we have $P_l/P_L \approx 0.015$, i.e., at a solar-cell efficiency of $\eta \approx 20\%$, the loss to the series resistance of the p^+ layer amounts to 0.3%.

We now find achievable characteristics of the island cell. For $V_m = 0.546$ V, $d/\tau + s_d + s_{0n} = 660$ cm/s (see [1] and Section 2), $d/\tau + s_{di} + s_{0n} = 10$ cm/s, it follows from formulae (39) and (40) that $V_{lim} = 0.649$ V and $P_{lim}/P_m = 1.198$. Further, we obtain $u \approx 0.237$, $V_{opt} = 0.642$ V from formulae (38) and (41a), at the island and material parameters $r_{0,\,opt} = 5 \times 10^{-4}$ cm, $s_{0pM} = 400$ cm/s, $D = 30$ cm^2/s. Then, we obtain $I_{opt}/I_m \approx 0.981$, $P_{opt}/P_m \approx 1.152$ (to which corresponds $\eta \approx 22.4\%$) from formula (37), and $A \approx 1.9 \times 10^{-4}$ cm^2 from formula (41b).

For the ellipsoidal surface of an n^+ island, formulae (36)–(40) are constant, and the coefficients $1/4\pi$, $1/8\pi$, and 8π in formulae (35a), (35b), and (41b), are replaced correspondingly with $1/2f(\beta)$, $\pi g(\beta)/4f^2(\beta)$, and $4f^2(\beta)/\pi g(\beta)$, and formula (41à) includes also the coefficient $f(\beta)/\pi g(\beta)$. For example, for a flat n^+ island ($\beta = 0$), the factors $4/\pi$ for $r_{0,\,opt}$ and $8/\pi^2$ for A enter into the right-hand sides of formulae (41a) and (41b), and the previously found values u, I_{opt}, V_{opt}, P_{opt} are invariable if we take $r_{0,\,opt} = 6.4 \times 10^{-4}$ cm and $A = 1.5 \times 10^{-4}$ cm^2.

The characteristics of island-structured solar cells with various values of parameters $r_{0,\,opt}$ and $d/\tau + s_d + s_{0n}$ are listed in the Table.

7. DISCUSSION OF RESULTS

Calculating the characteristics of an island-structured solar cell, we compared them with the measured characteristics of a conventional cell and, thus, implicitly took into account the effect of series resistance and recombination in the space-charge region of the p–n junction decreasing the filling factor and the efficiency of the solar cell. We note that the values of this efficiency calculated above are probably somewhat underestimated because the island-containing cells have a photoresponse in the short-wavelength region of the spectrum (due to the lack of an n^+ layer and decreased SR on the face surface). In addition, replacement of the n^+ layer with small n^+ islands actually eliminates the effect of recombination in the space-charge region.

The repetition period of islands along the axes X and Y on the rear side should be identical. Then, for the set series resistance of the p^+ layer, the distance between islands can be made largest. The repetition period of islands along axes X and Y (a and b) on the face side should be chosen to be different. The point is that, in the approximation used by us, the solar-cell efficiency depends only on their product $A = ab$. Therefore, the distance a between islands along bus-bars should be chosen less than the distance in the perpendicular direction b. This fact enable us to open up the face surface as much as possible for light at the smallest technologically allowable width of the metal bus-bars.

Our analysis showed that the solar-cell efficiency increases due to a large decrease in the ESR (from ~100 cm/s to ~1 cm/s) if we lower the doping level of the p^+ layer from ~(10^{20}–10^{21}) cm^{-3} to ~10^{18} cm^{-3}. Such an optimum concentration exists because, with an increase in concentration by these two–three orders of magnitude, the potential barrier between the base and insulator increases to a lesser extent (because of band-gap narrowing) than the SR (it increases by two–three orders of magnitude). In this case, the ESR increases also because of increasing rate of Auger recombination.

APPENDIX

Influence of the Shape of the n^+ Regions

Let us generalize our results to the case of the ellipsoidal surface of an n^+ island. In the case of ~~a flattened~~ ellipsoid of rotation, the ellipsoidal surfaces in the coordinates σ, τ, and φ, such that $\sigma \geq 0$, $-1 \leq \tau \leq 1$, and $0 \leq \varphi < 2\pi$ have the form

$$\frac{x^2 + y^2}{t^2(1+\sigma^2)} + \frac{z^2}{t^2\sigma^2} = 1. \quad \text{(A.1)}$$

For the surface of the p–n junction representing a circle of radius r_0 in the plane $z = 0$ and passing at $x = 0$ and $y = 0$ through point $z = \beta r_0$, where $\beta < 1$, t and σ_s on the contact surface are expressed in terms of r_0 and β by the formulae $\beta r_0 = t\sigma_s$ and $r_0 = t\sqrt{1+\sigma_s^2}$. From these relations, we obtain

$$t = r_0\sqrt{1-\beta^2}, \quad \sigma_s = \beta/\sqrt{1-\beta^2}. \quad \text{(A.2)}$$

The boundary condition of the type of (15a) on the ellipsoidal contact has the form

$$n(\sigma_s = 0), \quad \text{(A.3)}$$

and, instead of Eq. (17), the following equation is solved here at $\sigma_s \leq \sigma < \infty$:

$$\Delta n \equiv \frac{1}{t^2(\sigma^2+\tau^2)}\left\{\frac{\partial}{\partial\sigma}\left[(1+\sigma^2)\frac{\partial n}{\partial\sigma}\right] + \frac{\partial}{\partial\tau}\left[(1-\tau^2)\frac{\partial n}{\partial\tau}\right] + \frac{\sigma^2+\tau^2}{(1+\sigma^2)(1-\tau^2)}\frac{\partial^2 n}{\partial\varphi^2}\right\} = 0. \quad \text{(A.4)}$$

Assuming that the electron concentration depends only on the coordinate σ, we have from (A.4) $\partial n/\partial\sigma = C(1+\sigma^2)$, where C is an arbitrary constant. From here and condition (A.3), we obtain

$$n(\sigma) = C(\arctan\sigma_s^{-1} - \arctan\sigma^{-1}). \quad \text{(A.5)}$$

Using the expression for the gradient of the scalar function in orthogonal coordinates

$$\nabla\Phi = \pm\left(\frac{1}{\sqrt{g_{11}}}\frac{\partial\Phi}{\partial x^1}\mathbf{i}_1 + \frac{1}{\sqrt{g_{22}}}\frac{\partial\Phi}{\partial x^2}\mathbf{i}_2 + \frac{1}{\sqrt{g_{33}}}\frac{\partial\Phi}{\partial x^3}\mathbf{i}_3\right),$$

where the diagonal elements of the Jacobian in the coordinates under use are

$$g_{\sigma\sigma} = t^2(\sigma^2+\tau^2)/(1+\sigma^2),$$

$$g_{\tau\tau} = t^2(\sigma^2+\tau^2)/(1-\tau^2),$$

$$g_{\varphi\varphi} = t^2(1+\sigma^2)/(1-\tau^2),$$

and formula (A.5), we obtain the following expression for the flux density of electrons normal to the surface of the p–n junction located at $\sigma = \sigma_s$:

$$D\Delta n|_{\sigma=\sigma_s} = D\sqrt{1+\sigma_s^2}/(t\sqrt{\sigma_s^2+\tau^2})\left.\frac{\partial n}{\partial\sigma}\right|_{\sigma=\sigma_s}\mathbf{i}_\sigma = DC/[t\sqrt{(\sigma_s^2+\tau^2)(1+\sigma_s^2)}]\mathbf{i}_\sigma. \quad \text{(A.6)}$$

If the surface S is a part of surface $x^k = \text{const}$ on which x^i and x^j form the system of orthogonal coordinates, $\mathbf{dS} = \sqrt{g_{ii}g_{jj}}\,dx^i dx^j\mathbf{i}_k$. From here it follows that the small element of the surface $\sigma = \sigma_s$ in coordinates of a flattened ellipsoid is

$$\mathbf{dS}_\sigma = t^2\sqrt{(\sigma_s^2+\tau^2)(1+\sigma_s^2)}\,d\tau\,d\varphi\,\mathbf{i}_k. \quad \text{(A.7)}$$

Thus, the flux passing through the surface of the p–n junction is

$$J = -\int\limits_{S_{\text{cont}}} dS_\sigma d\nabla n|_{\sigma=\sigma_s} = -\int_0^1 d\tau\int_0^{2\pi} d\varphi\,DCt = -2\pi DCr_0\sqrt{1-\beta^2}. \quad \text{(A.8)}$$

From formulae (A.5) and (A.8), we obtain the equation relating the electron flux and their concentration at values of σ such that $r_0 \ll 6r_0\sqrt{1-\beta^2}/\beta \ll a, b, d, L$ {see expression (20)}:

$$0 = J + 2\pi Dr_0\sqrt{1-\beta^2}\,n(0)/\arctan\sigma_s^{-1} = J + f(\beta)Dr_0n(0),$$

where function $f(\beta)$ is determined by the expression

$$f(\beta) = 2\pi\sqrt{1-\beta^2}/\arctan(\sqrt{1-\beta^2}/\beta). \quad \text{(A.9)}$$

As a result, formula (21) now is replaced with the following expression:

$$-D\partial n(z)/\partial z|_{z=0} = -[f(\beta)Dr_0/ab + s_{0n}]n(0), \quad \text{(A.10)}$$

whence it follows that the generalization of formulae (23) and (24) for the ellipsoidal surface of the n^+ island is attained by replacing 2π with the value of $f(\beta)$ determined using formula (A.9) in them. In particular, formula (24) then takes the form

$$I_{Li} = Ai_L/[1 + A(d/\tau + s_{0n} + s_d)/f(\beta)r_0D], \quad \text{(A.11)}$$

while condition (25) should be rewritten as follows:

$$A(d + s_{0n}\tau + s_d\tau)/[f(\beta)L^2] \ll r_0 \ll a, b. \quad \text{(A.12)}$$

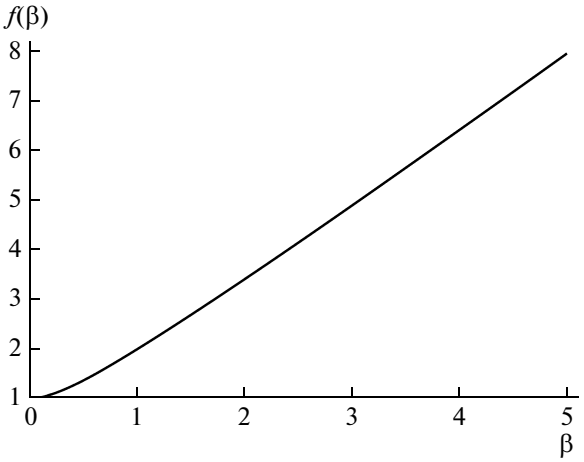

Fig. 1. Function $f(\beta)$.

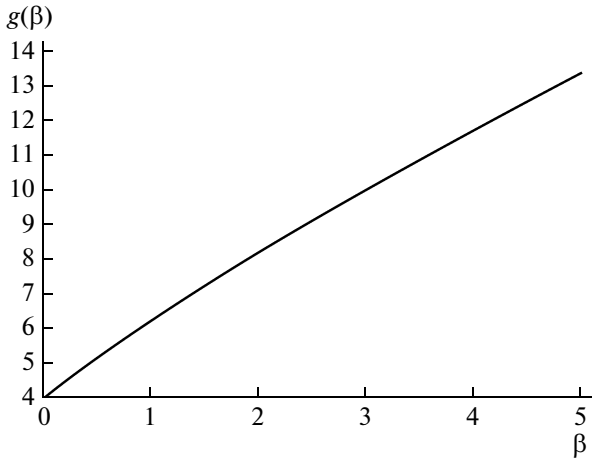

Fig. 2. Function $g(\beta)$.

We find the area of the surface of the n^+ island, S_{cont} using formulae (A.2) and (A.7):

$$S_{\text{cont}} = \int\limits_{S_{\text{cont}}} dS_\sigma = t^2\sqrt{1+\sigma_s^2}\int\limits_0^1 d\tau\sqrt{\sigma_s^2+\tau^2}\int\limits_0^{2\pi} d\varphi = \pi r_0^2 g(\beta), \quad \text{(A.13)}$$

where $g(\beta)$ is determined by the expression

$$g(\beta) = 1 + \beta^2\ln(1+\sqrt{1-\beta^2}/\beta)\sqrt{1-\beta^2}. \quad \text{(A.14)}$$

Now the expression for the dark current I_{di} {see (27)} takes the form

$$I_{di} \approx -eA\left(\frac{d}{\tau} + s_{0n} + s_d + \frac{2\pi g(\beta) r_0^2}{A} s_{0p}\right) \times n_{p0}(e^{eV/T} - 1). \quad \text{(A.15)}$$

However, if the surface of the n^+ island represents an extended ellipsoid of rotation, we again obtain formulae (A.10) and (A.13) from calculations similar to those carried out above with the only difference being that the functions $f(\beta)$ and $g(\beta)$ now are determined at $\beta \geq 1$ and, instead of formulae (A.9) and (A.14), they are described by new formulae:

$$f(\beta) = 2\pi\sqrt{\beta^2-1}/\ln(\beta+\sqrt{\beta^2-1}), \quad \text{(A.16)}$$

$$g(\beta) = 1 + \beta^2\arctan\sqrt{\beta^2-1}/\sqrt{\beta^2-1}. \quad \text{(A.17)}$$

The function $f(\beta)$ monotonically increases from 4 (at $\beta = 0$) to 2π (at $\beta = 1$), and $f(\beta) \longrightarrow \infty$ at $\beta \longrightarrow \infty$. The function $g(\beta)$ monotonically increases from one (at $\beta = 0$) to two (at $\beta = 1$), and $g(\beta) \longrightarrow \infty$ at $\beta \longrightarrow \infty$. The plots of functions $f(\beta)$ and $g(\beta)$ are shown in Figs. 1 and 2.

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